1. (a) Solve for x in the equation: $\frac{16^x-4^x}{4^x+2^x}=5(2^x)-8$

(b) Solve the equations

i). $\frac{x+4z}{4}=\frac{y+z}{6}=\frac{3x+y}{5}$ and $4x+2y+5z=30$

ii). $8^{x-y}=4^{x+y}$ and $5^{x^2-y^2}=15625$

2. Solve the inequality ;

(i) $\frac{x+1}{2x-1}\geq\frac{1}{x-3}$

(ii) $\left|\frac{2x-4}{x+1}\right|\leq 4$

(iii) $(0.8)^{-2x}>16$

3.(a) Solve the equation $\sqrt{3-x}-\sqrt{7+x}=\sqrt{16+2x}$

(b) The nth term of a series is $h_n=x(3)^n+ay+z$. Given that $h_1=4, h_2=13$ and $h_3=46$. find the values of x, y and z.

4. (a) In the expansion of $(3x+2)^n$ in ascending powers of x, the coefficient of x^{11} is a quarter that of x^{12}. Determine the value of n.

(b) Express $\frac{2x^2-5x+7}{(4x^2-9)(x+2)}$ in partial fractions. Hence expand $\frac{2x^2-5x+7}{(4x^2-9)(x+2)}$ in ascending powers of x up to the term containing x^2.

5.(a) Determine the values of p and q if the polynomial $f(x)=x^3+px^2+qx-18$ is divisible by $(x+3)^2$.

(b) Show that $x=2$ is a root of the equation $x^3-x-6=0$. Deduce the values of $\alpha+\beta$ and $\alpha\beta$ where α and β are the roots of the other equation. Hence find a quadratic equation whose roots are α^2 and β^2.

(c) Given that the roots of the equation $x^2+6x+c=0$ differ by $2n$, where n is real and non – zero. Show that $n^2=9-c$. Given that the roots also have opposite signs, find the set of possible values of n.

6.(a). In the expansion of $\left(2^x-\frac{1}{4^x}\right)^n$, the sum of the binomial coefficients in the first and the second term is equal to 32, and the second term of the expansion is 7 times as large as the first. Find x

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(b). If α and β are roots of roots of the quadratic equation

$$x^2 - 4\sqrt{2}\,\lambda x + 2\lambda^4 - 1 = 0 \text{ and } \alpha^2 + \beta^2 = 66.$$

(i) Determine the positive value of λ

(ii) Determine the value $\alpha^3 + \beta^3$.

7. Prove by mathematical induction that:

(i) $\sin\theta + \sin 3\theta + \sin 5\theta + \cdots + \sin(n-1)\theta = \frac{\sin^2 n\theta}{\sin\theta}$

(ii) $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$

(iii) $\sin(x + 180^0 n) = (-1)^n \sin(x)$. For all n.

8. (a) Given that $w = -1 + i\sqrt{3}$, find the value of the real number μ such that $arg(w^2 + \mu w) = \frac{5\pi}{6}$.

(b) Express $w = (-1 + i\sqrt{3})^8$ in the form $a + ib$. Hence or otherwise find the cube root of w.

9. (a) If $z = x + iy$, show that the locus of $arg\left(\frac{z-1}{z-i}\right) = \frac{\pi}{6}$ is a circle. Find its centre and radius.

(b) Z and $\bar{Z}$ are conjugate complex numbers. Find the values of Z that satisfy the equation $3Z\bar{Z} + 2(Z - \bar{Z}) = 39 + 12i$

(c) Find the square root of $5 + 12i$

10(a) If $z = x + iy$, solve fort x and y in the equation $z^2 + 1 = 0$

(b) Solve for real numbers a and b in the equation $\frac{5}{a+ib} + \frac{3+4i}{2+i} = 4 + 2i$

(c) Solve the simultaneous equations for z_1 and z_2 :

$(2 + 3i)z_1 - 3z_2 = i$ and $4z_1 + 3iz_2 = 6 + 5i$

11. (a) Use Demoivre's theorem to prove that the complex number $\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n$ is always real and hence find the value of the expression when n = 6.

(b) If $-4 - 3i$ is one root of the equation $Z^4 - 4Z^3 - 4Z^2 - 4Z + 925 = 0$, Determine the other roots of the equation.

12(a) Three consecutive terms of a geometric series have product 343 and sum $\frac{49}{2}$. Find the numbers.

(b) The product of the third and sixth terms of the Arithmetic progression is 406. The quotient of the division of the ninth term by the fourth term of the progression is equal to 2, and the remainder is -6. Determine the first term and the common difference of the progression.

(c) A man invests \$1000 at the beginning of each year for ten years. The rate of compound interest is 9% per annum. Calculate the total value of the investment at the end of the ten full years.

THEME 2 : TRIGONOMETRY(1A, 1B)

11. (a) Solve the equation $4^{\sin 2x+2\cos^2 x}+4^{1-\sin 2x+2\sin^2 x}=65$ for $0^\circ \le x \le 360^\circ$.
   (b) Prove that $\cos 3\theta+\sin 3\theta=(\cos\theta-\sin\theta)(1+2\sin 2\theta)$
   (c) If $2a\cos 2x+b\sin 2x+2c=0$ where $a\neq 0$ and $a\neq c$.
      i). Find an equation $\tan x$
      ii). State the sum and product of the roots of this equation. $\tan x_1$ and $\tan x_2$ and hence deduce that $\tan(x_1+x_2)=\frac{b}{2a}$.

12. (a) Solve (i) $\sin 3x+\sin^3 x=\frac{3\sqrt{3}}{4}\sin 2x$ for $0^\circ \le x \le 360^\circ$.
      (ii) $5\sin(x+60^\circ)-3\cos(30^\circ+x)=4$ for $0\le x\le 2\pi$
   (b) Find all the angles between 0° and 180° for $\frac{2}{\cos^2 2x}-4=3\tan 2x$

13(a) Solve the equation:
   (i) $8\cos^4 x=3\cos 4x+\cos 2x+4$ for $0\le x\le 2\pi$.
   (ii) $81^{\sin^2 2x}+81^{\cos^2 2x}=30$ where $0^\circ \le x\le 360^\circ$
   (iii) $\sqrt{\cos^2 2\theta+\left|\sin\left(2x-\frac{3}{2}\pi\right)\right|+\frac{1}{4}}=\cos\left(\frac{20}{12}\pi\right)$ for $0^\circ\le\theta\le 360^\circ$
   (iv) $\cos 3x-\cos 2x=\sin 3x$
   (v) $\cos x+\cos 3x+\cos 5x+\cos 7x=0$
   (b) Find the maximum and minimum values of $\frac{\sqrt{2}}{\cos a-\sqrt{2}\sin a}$

14(a) Prove that: $\frac{3\sin\theta+\sin 2\theta}{1+3\cos\theta+\cos 2\theta}=\tan\theta$, hence, solve the equation

$$\frac{3\sin\theta+\sin 2\theta}{1+3\cos\theta+\cos 2\theta}+\frac{1}{\cos^2\theta}=2, \text{ for } 0^\circ\le\theta\le 360^\circ.$$

(b) Prove that $\sin 4\phi=\frac{4\tan\phi(1-\tan^2\phi)}{(1+\tan^2\phi)^2}$.

(c) Solve the equation $\tan^{-1}(2x+1)+\tan^{-1}(2x-1)=\tan^{-1}2$

THEME 3 : ANALYSIS (3A, 3B)

15. a) Find the first three terms in the expansion of $\frac{x^2+4x+3}{\sqrt[3]{(1+\frac{3}{2}x)}}$ in ascending powers of x. Hence evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{x^2+4x+3}{\sqrt[3]{(1+\frac{3}{2}x)}}\,dx$

(b) Prove that $\int_0^{\frac{1}{2}}\frac{1+\sqrt{3}}{[(x+1)^2(1-x)^6]^{\frac{1}{4}}}\,dx=2$

16.(a) Find $\int\frac{x}{x^3+1}\,dx$ $\longrightarrow$ $\int\frac{1}{(x+1)(x^2-x+1)}\,dx=$

$1=A(x^2-x+1)+(Bx+C)(x+1)$

(b) Given that $f(x)=\frac{1}{(1-x)(1+2x)}$.

i). Express $f(x)$ in partial fractions.

ii). Determine the first three terms of the expansion of $f(x)$ in ascending powers of x. Hence find the coefficient of x^n and state the range of values of x for which the expansion is valid.

iii). Using partial fractions, determine $f'(x)$.

iv). Evaluate $\int_0^1 f(x)dx$.

17. (a) If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ prove that $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

(b) Given that $y=\frac{(x-3)^2}{(x-1)(x-5)}$,

i). Show that for real values of x, y cannot lie between 0 and $\frac{3}{4}$ hence determine the turning points

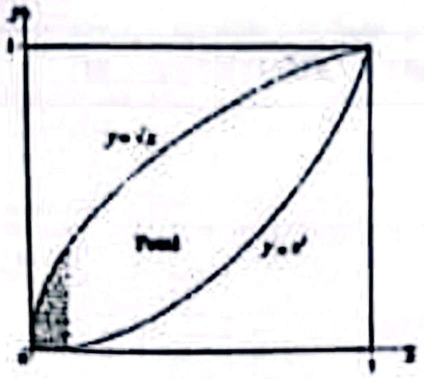
ii). State all the asymptotes and intercepts of this curve

iii). Sketch the curve.

18(a) Differentiate each of the following with respect to x:

(i) $x^{\tan x}+(\sin x)^{\cos x}$ (ii) $\frac{e^{x^2}\sqrt{\sin x}}{(2x+1)^3}$ (iii) $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ (iv) $\sqrt{\frac{(x+2)^3}{x-1}}$

(b) The diagram below shows the design of a petal drawn on a square tile of length 1m



The design may be modelled by the finite region enclosed by the curves $y = \sqrt{x}$ *and* $y = x^2$ where *x and y* are lengths measured in metres, determine the area of the petal.

19 (a) Differentiate

i). $y = \sqrt{\sin x}$ from first principles

ii). $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

(b) When expanded in ascending powers of x, the Maclaurin's expansion of $\ln(e^{2x} + e^{-2x}) = a + bx^2 + cx^4$. Determine the values of a, b *and* c.

20. Sketch, on the same axes, those parts of the curve $y = 16 - x^2$ and the line $y = 6x$ which lie in the same quadrant. Shade the area which satisfies $y \leq 16 - x^2, y \geq 6x$ *and* $x \geq 0$. Find the volume generated when this area is rotated completely about the $x-axis$ eaving your answer as a multiple of π

21. (a) The area enclosed by $y = x^2 - 6x + 18$ and $y = 10$ is rotated about the y-axis, find the volume generated take an element of area parallel to the x-axis of length $(x_2 - x_1)$;
Express the typical element of volume in terms of y by using the fact that x_1 and x_2 are the roots of $x^2 - 6x + (18 - y) = 0$

(b) A garden water tank is in the shape of a cuboid with a square base of sides 1.4m, and height 2.8m. There is a small hole at the bottom of the tank from which water is leaking. After *t minutes*, when the depth of water is *h metres*, the rate of escape of water is $40\sqrt{h}\ m^3min^{-1}$.

Show that $\frac{dh}{dt} = -\frac{1000}{49}\sqrt{h}$. If the tank is full initially, how long will it take to empty?

39. (a) Solve the differential equations below;

(i) $(1+x)\frac{dy}{dx} = 1 - \sin^2 y$ $for\ which\ y = \frac{\pi}{4}\ when\ x = 0$

(ii) $\frac{dy}{dx} + 2y = e^{-2x}(x^3 + x^{-1}),\ for\ y(1) = 0$

(iii) $(x+y)\frac{dy}{dx} = x^2 + xy + x + 1\quad if\ y = v - x$

(iv) $x\frac{dy}{dx} - 2y = x^3 \ln x;\quad if\ y = 2\ when\ x = 1$

(v) $\frac{dy}{dx} = \frac{y(x+2y)}{x(y+2x)},\ if\ x \neq 0$

(b) The population of a certain country has grown at a rate proportional to the number of people in the country. At present, the country has 80 million inhabitants. Ten years ago, it had 70 million. Assuming that this trend continues, determine

(i) An expression for the approximate number of people living in the country at any time t.

(ii)The approximate number of people who will inhabit the country at the end of the next ten – year period.

THEME 4 : VECTORS (1A, 1B)

23(a) A right circular cone has vertex at the point $(4,-5,3)$ and the centre of the base at the point $(0,1,-1)$. Write down the

(i) equation of the axis of the cone

(ii) equation of the plane containing the base. If the line $\frac{x-4}{3}=\frac{y+5}{-8}=\frac{z-3}{2}$ is the generator of the cone, find the coordinates of the point where this generator meets the base. and deduce that the volume of the cone is $6\pi\sqrt{17}$

(b) If the position vectors of points A and B are $2\mathbf{i}+4\mathbf{j}+6\mathbf{k}$ and $-3\mathbf{i}+2\mathbf{j}+8\mathbf{k}$ respectively, find the position vector of the point P which divides $\mathbf{AB}$ externally in the ratio $5:3$.

24 (a) A plane contains points M(4, -6, 5) and N(2, 0, 1). A perpendicular to the plane from the point P(0, 4, -7) intersects the plane at point Q. Find the Cartesian equation of the line $\overline{PQ}$.

(b) The points A, B and C have position vectors $\begin{pmatrix} 4 \\ 10 \\ 5 \end{pmatrix}, \begin{pmatrix} 6 \\ 8 \\ -2 \end{pmatrix}, and \begin{pmatrix} 1 \\ 10 \\ 3 \end{pmatrix}$ respectively. If A, B and C are the vertices of a triangle show that angle ABC is a right angle.

(c) OPQR is a trapezium with $\mathbf{OP} = 3\mathbf{p}$ and $\mathbf{OR} = \mathbf{r}$ and $\mathbf{RQ} = 2\mathbf{p}$. S is a point on PQ such that $PS : SQ = 3:1$ and OS crosses RP at T. If $\mathbf{PT} = m\mathbf{PR}$ and $\mathbf{OT} = n\mathbf{OS}$, find the values of m and n.

25 (a) The line L passes through the points A and B whose position vectors are $3i - j + 2k$ and $-i + j + 9k$ respectively. Find in vector form, the equation of the line L.

(b) The line L_2 has the equation $r = (8i + j - 6k) + \lambda(i - 2j - 2k)$ where λ is a scalar parameter.

(i) show that the lines L_1 and L_2 intersect.

(ii) Determine the position vector of the point of intersection.

(c) Show that the lines $\mathbf{r} = (-2\mathbf{i} - 5\mathbf{j} - 11\mathbf{k}) + \lambda(3\mathbf{i} + \mathbf{j} + 3\mathbf{k})$, $\mathbf{r} = (8\mathbf{i} + 9\mathbf{j}) + t(4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ intersect, hence, find the position vector of their point of intersection. Find also the Cartesian equation of the plane formed by these two lines.

26. (a) Find the angle between the planes $r.(2i - j - 3k) = 10$ and $r.(i + 3j - 2k) = 16$.

(b) Find a vector equation of the plane passing through the points $A(1,0,0), B(2,-6,1)$ and $C(-3,0,4)$. Hence determine the coordinates of the point of intersection of the plane ABC and the line $r = 2\mu i + (1 - 5\mu)j + (\mu - 2)k$.

THEME 4 : GEOMETRY (1A, 1B)

27. (a) The three straight lines $y = x$, $2y = 7x$, and $x + 4y - 6 = 0$, form a triangle. Find the coordinates of the point of intersection of the

medians of the triangle.

(b) The vertices of a triangle are A(1, 6), B(-5, 2) and C(3, 4) respectively. Find the coordinates of its:

I. Orthocentre
II. Circumcentre
III. Centroid

(c) A straight line MN of length 10cm is free to move with its ends on the axes. Find the locus of a point M on the line at the distance of 3cm from the end on the x – axis.

28. (a) A conic section has parametric equations $x = 12\sin\theta$ and $y = 9\cos\theta$. Show that the conic section is an ellipse and find its eccentricity.

(b) The line $x + cy + d = 0$ touches the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ Show that $d^2 = a^2 + b^2c^2$, hence determine gthe equation of the four common tangents to two ellipse $4x^2 + 16y^2 = 56$ and $3x^2 + 23y^2 = 69$.

(c) Express $r^2 = \operatorname{cosec}2\theta$ in Cartesian form.

29 a) The line $y = ax + 3$ is a tangent to the parabola $y^2 = 4ax$. find the value of a.

b) A tangent to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ meet the directrix at point Q.

Point R is the foot of the perpendicular from the vertex to the tangent.

i) Show that SP and SQ are perpendicular.
ii) Find the locus of the mid-point of OR

(c) Point $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on the parabola $y^2 = 4ax$. Find the locus of the midpoint of the chord PQ for which $pq = 2a$.

30a) Find the equation of a circle through the points $A(0, 1)$ $B(2, 3)$ and $C(0, 5)$

b) Given that $x = 1 + \sqrt{2}\sin\theta$ and $y = -3 + \sqrt{2}\cos\theta$.

i) Show that x and y represents a circle. Hence its state the centre and radius

ii) Find the equation of the common chord of intersection of the circles

2. The table below represents marks for ten students in Biology and Chemistry of a certain school.

Biology(x)	40	90	54	32	80	65	55	48	55	30
Chemistry(y)	68	40	47	64	55	41	62	76	74	80

a) Plot a scatter diagram for the data, draw a line of best fit and comment on the relationship between the two subjects

b) Estimate the Biology mark, if the Chemistry mark was 60.

c) Calculate the rank correlation coefficient and comment at 5% level of significance.

Simple data — $S.D = \sqrt{\frac{n}{n-1}\left(\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2\right)}$

3. The weights in kg of 65 boxes were as follows.

Weight(kg)	Number of boxes
120 – 124	3
125 – 129	10
130 – 134	12
135 – 139	18
140 – 144	15
145 – 149	6
150 - 154	1

Grouped data $\sqrt{\frac{\sum f}{\sum f - 1}\left[\frac{\sum fx^2}{\sum f} - \bar{x}^2\right]}$

Mean: $\frac{\sum fx}{\sum f}$

Standard deviation $= \sqrt{Var(x)}$

$Var(x) = \frac{\sum fx^2}{\sum f} - (\bar{x})^2$

(a) Calculate the:

(i) Mean weight (ii) Standard deviation

(b) Construct a cumulative frequency curve for the data and use it to estimate the limit within which the weights of the middle 40% of the boxes lie.

M.40% $P_{30} = \frac{30}{100} \times 65 = 19.5$ / $P_{70} = \frac{70}{100} \times 65 = 45.5$

4. (a) In a certain restaurant, 40% of the customers' order for local food. If a customer orders Local food, the probability he will take a drink is 0.6. If he does not order food, the probability that he will take a soft drink is 0.3. Determine the probability that a customer picked at random will order;

(i) Local food and a soft drink (ii) a soft drink

(b) Box P contains 3red balls and 4 Green balls while box Q contains 3 red balls and 2 green balls. A box is drawn randomly and one ball is randomly drawn from it. Find the probability that the ball:

i). is red

ii). came from box P, given that it is red.

(c) A biased coin is tossed 12 times; the coin is such that the ratio of the head to the tail to land on top is 1:3 find the probability of getting:

(i) utmost 4 heads.
(ii) between 6 to 10 heads.

5. X is a random variable such that:

$$f(x) = \begin{cases} 2\mu(x+1) : & -1 \le x \le 0 \\ \mu(2-x) , & 0 \le x \le 2 \\ 0 : & elsewhere \end{cases}$$

(a) (i) Sketch the p.d.f. $f(x)$.

(ii) Determine the value of the constant, μ.

(b) Find the:

(i) Mean of X, and the standard deviation, δ.
(ii) cumulative distribution function $F(x)$.

6. (a) Given that M and N are events such that $P(\overline{M} \cap N) = x$, $P(M \cap \overline{N}) = z$ and $P(M \cap N) = y$. If $P(M) = 0.62$, $P(N) = 0.44$ and $P(\overline{M} \cap \overline{N}) = 0.22$, find the values of x , y and z.

(b) Two events A and B are such that $P(A'/B') = \frac{2}{7}$ and $P(B) = \frac{2}{3}$. Find the :

(i) $P(A \cup B)$
(ii) $P(A \cup B')$.

7. The continuous random variable X has a cumulative function $F(x)$. where:

$$F(x) = \begin{cases} 0 & ; x \le 1 \\ \frac{(x-1)^2}{12} & ; 1 \le x \le 3 \\ \frac{1}{24}(ax - bx^2 - 25) & ; 3 \le x \le 7 \\ 1 & ; x \ge 7 \end{cases}$$

Determine the :

a) Values of a and b.
b) Probability density function, $f(x)$ and sketch it.
c) Determine the
i). Mean, $E(x)$
ii). Median
iii). 4th decile
iv). 90th percentile

v). Semi – interquartile range

8. (a)The ideal size of first – year class at a particular college is 150 students. The college, knowing from past experience that, on average, only 30% of those accepted for admission will actually attend, uses a policy of approving the applications of 450 students. Compute the probability that more than 150 first year students attend this college.

(b) 1 in 8 of the residents of a particular city were known to own cars. Find the approximate probability that at least 15 of a random sample of 100 of the city population will own cars.

9. (a) A ship A is streaming north at a speed of $12kmhr^{-1}$ and ship B is streaming due East at a speed of $16kmhr^{-1}$, given that initially 2km on a bearing of 330^0 from A. If they continue streaming with these speeds in these directions, determine
   i). Time when they are nearest each other.
   ii). Shortest distance.

   (b) At noon, particles A and B have the following position and velocity vectors.

Particles	Position vector	Velocity vector
A	$(10i + 5j + 8k)km$	$(-2i + 4j - k)kmhr^{-1}$
B	$(2i - j + 6k)km$	$(2i + 7j)kmhr^{-1}$

   i). Find the displacement of B relative to A at any time t seconds.
   ii). If A and B collide, find the time and positions of collision.

10. (a) A non - uniform ladder PQ of length 12m and mass 30kg is in limiting equilibrium with its lower end P resting on a rough horizontal ground with the coefficient of friction μ and the upper – end Q resting against a rough vertical wall with the coefficient of friction 0.2. The weight of the ladder acts at R, where PR = 4m, with the ladder making an angle 66^0 with the horizontal. A straight horizontal string connects P to a point at the base of the wall. If a man 90kg climbs up to the top of the ladder and the tension in the string is 126N, find the:
   i). Reaction at P and Q
   ii). Value of μ.

   (b) A non-uniform ladder AB whose centre of gravity is 2 m from end A is of length 6 m and weight W. The ladder is inclined at angle of θ to the vertical with its end B against a rough vertical wall and end A on a rough horizontal ground with which the coefficients of friction at each point of contact is μ. If the ladder is about to slip when a man of weight 5W ascends two–thirds of the way up the ladder, show that $\tan\theta = \frac{18\mu}{11-7\mu^2}$.

11. A car of mass 1 tonne has a maximum speed $15ms^{-1}$ up a slope whose inclination to the horizontal is $\sin\theta = 0.2$. There is a constant frictional

resistance equal to one - tenth of the weight of the car.

a) Find the maximum

i). Power output of the car

i). Speed of the car on a level ground

b) If the car descends the same slope with its engine working at half its maximum power, find the acceleration of the car at the moment when its speed is $30ms^{-1}$.

12. (a) A truck of mass 1200kg tows a van of mass 300kg up a hill inclined at 1 in 100 against a constant resistance of 0.2N per kg. Given that the truck moved at a constant speed of $1.5ms^{-1}$ for 5 minutes, calculate the:

i). Tension in the tow bar

i). Work done by the engine of the car during this time

ii).Total resistance, if the engine develops 15kW at a maximum speed of $72kmhr^{-1}$ on a level road

(b) A car traveling at $30ms^{-1}$ and has no tendency to slip on the truck of radius 250m banked at an angle β, when the speed in increased to $40ms^{-1}$ the car is on the point of slipping up the track. Determine the

i). Angle β.

i). Coefficient of friction

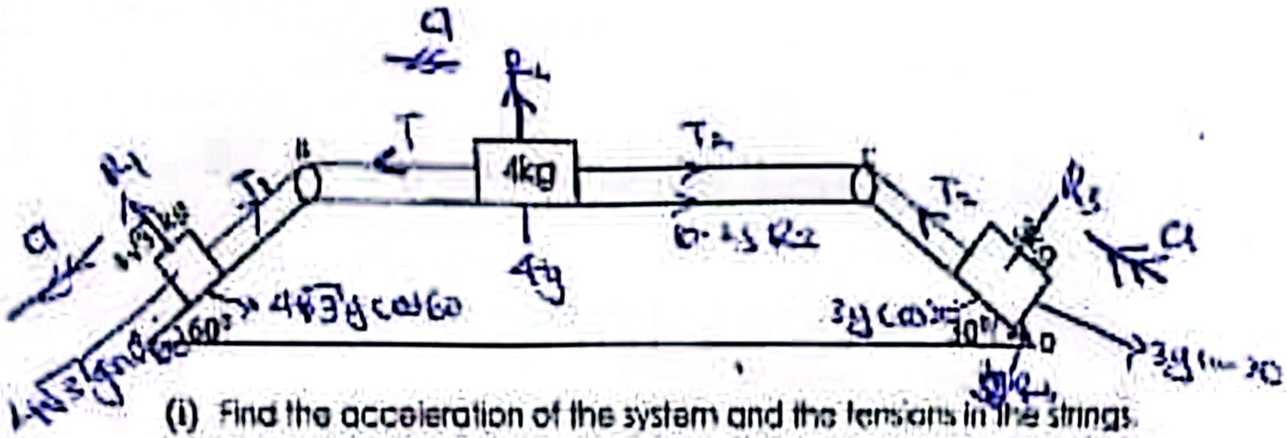
ii). Minimum velocity of the truck.

13. (a) A light inextensible string attached to a ceiling passes under a smooth movable pulley of mass 2kg and then over a smooth fixed pulley. A particle of mass 3kg hangs freely from the end of the string. All parts of the string not touching the pulley are vertical. If the system is released from rest, find the

i). Acceleration of the particle

ii). Tension in the string.

(b) The diagram below shows a 4kg mass on a horizontal rough plane with coefficient of friction 0.25. The $4\sqrt{3}$ kg mass rests on a smooth plane inclined at angles of $60°$ to the horizontal while the 3kg rests on a rough plane inclined at an angle $30°$ to the horizontal and coefficient of friction $\frac{1}{\sqrt{3}}$. The masses are connected to each other by light inextensible strings passing over light smooth fixed pulleys B and C.



(i) Find the acceleration of the system and the tensions in the strings.
(ii) Determine the work done against friction if each particle travels a distance of 0.5m

14. (a) A light elastic string of modulus λ and natural length $3a$ is fixed to two points on the same horizontal level at a distance $4a$ apart. Two particles of weight W are attached, one at each point of trisection. If the sloping string makes an angle θ with the horizontal in equilibrium, prove that the string extends by $\frac{2}{3}a(3-2\cos\theta)$, hence deduce that $\tan\theta = \frac{3W}{\lambda(3-2\cos\theta)}$.

(b) A and B are two fixed points on a smooth horizontal surface with AB, 2m. A body of mass 4kg lies on the line AB at a point P and is in equilibrium with a light string of natural length of 75cm and modulus 18N connecting it to B. Show that P is midway between A and B. If the body is pulled 20cm towards A and released, show that the subsequent motion is simple harmonic and find the maximum speed of the body during motion.

15. (a) A particle projected vertically upwards is $t = 0$ with a velocity U, passes a point at a height, h at $t = t_1$ and $t = t_2$. Show that $t_1 + t_2 = \frac{2U}{g}$ and $t_1 t_2 = \frac{2h}{g}$.

(b) Trials are being undertaken on a horizontal road to test the performance of an electrically powered car. The car has a top speed V. In a test run the car moves from rest with uniform acceleration a and is brought to rest with uniform retardation, r.

i. If the car is to achieve top speed during a test run, by using a velocity – time sketch, or otherwise, show that the length of the test run must be at least $\frac{V^2(a+r)}{2ar}$.

ii. Find the least time taken for a test run of lengths $\frac{2V^2(a+r)}{9ar}$ and $\frac{2V^2(a+r)}{3ar}$.

iii. Determine in terms of V, the average speed of the car for the test run described by the time $\frac{2V^2(a+r)}{3ar}$.

16. Two rigid light rods AB, BC each of length $\frac{3}{2}m$ are smoothly jointed at B and the rod AB is smoothly jointed at A to a fixed smooth vertical rod. The joint at B has a particle of mass 2kg attached. A small ring, of mass 1kg is smoothly jointed to BC at C and can slide on the vertical rod below A. The ring rests on a smooth ledge fixed to the vertical rod at a distance $\frac{\sqrt{3}}{2}m$ below A. The system rotates about the vertical rod with constant angular velocity 6 radians per second. Calculate the

a) Forces in the rods AB and BC.

b) Force exerted by the ledge on the ring.

17. A bus of mass 5tonnes freewheels down a slope of inclination $\sin^{-1}\left(\frac{1}{40}\right)$ to the horizontal at constant speed. Assuming that the non – gravitational resistances remain the same,

a) Find the rate at which the engine must work in order to drive the bus up the same incline at a steady of $12kmh^{-1}$.

b) If the power is suddenly increased to 10kW, determine the immediate acceleration of the bus.

THEME 3 : NUMERICAL METHODS

18(a) By sketching the graphs of $y = xe^x$ and $y = 1 - x$, show that the equation $xe^x + x - 1 = 0$ has root between $x = 0$ and $x = 1$, correct to 1dp.

(b) Using the initial approximation (x_0) from the graph above and the Newton Raphson method, find the root correct to 3 decimal places.

(c)(i) Use the trapezium rule with 6 strips to evaluate $\int_1^2 \log_5 3x\, dx$, correct to 4 significant figures.

(ii) Find the exact value of $\int_1^2 \log_5 3x\, dx$. Hence find the maximum possible error in your calculations in (i) above.

19 (a) The base and height of a triangle were measured as 11cm and 14cm with relative errors 0.03 and 0.04 respectively. Find the range within which the area of the triangle lies.

(b) The height and radius of a cylinder are measured as h and r with maximum possible errors Δ_1 and Δ_2 respectively. Show that the maximum percentage error made in calculating the volume is

$$\left(\left|\frac{\Delta_1}{h}\right| + 2\left|\frac{\Delta_2}{r}\right|\right) \times 100 .$$

(c) Given that the numbers $a = 1.8,\ b = 0.345$ and $c = 2.59$ all rounded off to the given number of decimal places. Find the maximum possible error in $\frac{a-b}{bc}$ and the limits within which the exact value of $\frac{a-b}{bc}$ lies.

20. (a) Jinja and Mukono are 66km apart. At 8:00am, Waiswa starts cycling towards Mukono from Mbikko town which is between Jinja and Kampala 2km away from Jinja. If at 8:00 pm, he has only covered 36km, estimate;

i). His distance from Mukono at 10:00pm.

ii). When he reaches Mukono.

(b) The table below is an extract from the tables of cosecθ^0

θ	15′	20′	25′	30′
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]